## Chapter 1: Measuring Up

## Introduction

If you haven't yet read the introduction to this book (pp. iii-x), please do so now.
Chemistry is the study of matter and how it changes. When you study chemistry, then, you have to be able to analyze matter and determine whether or not it is changing. How do you do that? Let's start with a definition:

$$
\underline{\text { Matter }} \text { - Anything that has mass and takes up space }
$$

So, if you are going to study matter and how it changes, you are going to have to figure out how to measure its mass as well as how much space it occupies. There will be other things you will have to measure as well. This is where we will start - how to measure things.

Now you might think you know a lot about measuring things already. You probably step on a bathroom scale every now and then to measure your weight. When you help out in the kitchen, you probably follow recipes, which require you to measure out specific amounts of ingredients. It is important, however, that you know exactly how to measure things and exactly what it is you are measuring.

## Measuring Distance

One way to measure how much space an object takes up is to measure its dimensions, like length, width, and height. This means measuring distance. I am sure you have already done this. Most students are interested in how tall they are, so they measure their height from time to time. How do you measure your height? Probably with a tape measure or a ruler. Even though you've had some experience measuring distance, it's important to make sure you are doing it correctly. Let's start with how to use a ruler.

How do you typically use a ruler? Most people in the United States use a ruler that is marked off in inches (abbreviated as "in"), such as the one shown below:


How would you use this ruler to measure, say, the length of a pencil? Here is how a scientist would do it:


Notice first that the pencil is not put at the left edge of the ruler. This is important. Even though the ruler's markings start there, it is not an accurate place for you to start your measurements, because the first mark usually isn't exactly on the edge. In addition, the edge can get banged up, so there isn't a clean start to it. As a result, it is best to start your measurement at an easy-to-see line that is not at the ruler's end. That's why one end of the pencil is lined up on the " 1 inch" mark of the ruler.

Now, to read how long the pencil is, we have to see how the ruler is marked off. Each number represents an inch, and there are 16 lines going from one inch mark to the next. That means the ruler is marked off in $16^{\text {th }}$ s of an inch. However, since there is space in between each $16^{\text {th }}$-inch mark, you can actually determine whether or not the end of the pencil hits one of those marks exactly or is somewhere in between two of the marks. Let me give you the illustration again so you can do that now:


If I look at the ruler, I see that the pencil goes from the 1 -inch mark to the $6^{\text {th }}$ mark past the 8 -inch mark. However, it actually goes past the $6^{\text {th }}$ mark but doesn't make it to the $7^{\text {th }}$ mark. So it is somewhere in between the $\frac{6}{16}$ mark and the $\frac{7}{16}$ mark. As a result, I would say it is at $\frac{13}{32}$, which is halfway in between $\frac{6}{16}$ and $\frac{7}{16}$. So I would say that the pencil is $7 \frac{13}{32}$ inches long.

Usually, we like to write measurements as decimals, so I would like to get rid of the fraction. If I divide 13 by 32 (which is what the fraction means), I get 0.40625 . So that means the pencil is 7.40625 inches long, right? Not quite. That measurement has too many digits in it. When a scientist reports a measurement, she has to be certain to report it to the proper precision. A ruler just isn't precise enough to measure the length of something to one hundred thousandth of an inch, which is the decimal place to which 7.40625 goes.

A ruler is, at best, precise to 0.01 inches. So in the end, you have to round your measurement to the hundredths place. Thus, the pencil is 7.41 inches long. Note that to round the number to the hundredths place, I had to drop the " 6 " that is in the thousandths place. When you are forced to drop a number that ranges from 5 to 9 , you add one to the number to its left. This is called "rounding up." So, in order to report the length of the pencil to the proper precision, I had to round the measurement up from 7.40625 inches to 7.41 inches.

This might sound like we are going to a lot of trouble to measure the length of a pencil, but when scientists make measurements, they have to be careful to report their answers properly. As a result, they have to know the precision of the instruments they are using. The precision of a ruler marked off in $16^{\text {th }}$ s of an inch is something you need to remember:

When using a ruler that is marked off in $16^{\text {th }}$ s of an inch, report your answer to hundredths of an inch.

## Using Different Units

Now let's suppose you want to measure the length of a pencil, but you don't live in the United States. Most likely, the ruler you have wouldn't use inches. Instead, it would probably use centimeters (abbreviated as "cm"), which is another unit for measuring length.

Unit - A quantity that describes the measurement being made

People in the Unites States often use English units, which were used in the British Empire for quite some time. However, most of the rest of the world uses metric units. While inches are part of the English unit system, centimeters are a part of the metric system. For most of this course, you will be using the metric system, because that's the one scientists generally use.

If you were using a metric ruler, then, how would you measure the length of the pencil? Well, the technique remains exactly the same:


You start with the pencil at the 1-centimeter mark, and you see how the ruler is marked off. There are ten marks between each centimeter, so the ruler is marked off in tenths of a centimeter. The end of the pencil goes past the $8^{\text {th }}$ mark after 19, but it doesn't reach the $9^{\text {th }}$ mark. As a result, it is somewhere between 19.8 and 19.9. It's not halfway in between, however. It is closer to 19.8 than 19.9, so we can say it is something like 19.82. You might say $19.81,19.83$, or 19.84 because your estimation might be a bit different from mine. That's fine. Of course, since the pencil starts at " 1 ," that means its length is 18.82 cm .

As a general rule of thumb, when you are reading a measuring device that has a scale to read, you can read the scale to a division below what is marked off. When we read the ruler that was marked of in $16^{\text {th }}$ s of an inch, we reported our answer to $32^{\text {nd }}$ s of an inch. In the same way, when you read a ruler marked off in tenths of a centimeter, you can report your answer to hundredths of a centimeter. Now, of course, that last digit will be an estimate. You don't know what it is exactly. You just have an idea of what it is. Nevertheless, it does make your measurement closer to correct than if you didn't report it. Thus, while it is only an estimate, it is still valuable. We will revisit this idea when we measure things other than distance.

It's important to realize, of course, that the actual length of the pencil is the same, even though it was measured with two different rulers. The pencil is both 7.41 inches and 18.82 centimeters. Each of those measurements represents the same length. They just do it using different units. Since you now know that both of those measurements represent exactly the same length, you should be able to answer the question below:

## Comprehension Check

1. Which is longer: an object that is 1 inch long or an object that is 1 centimeter long?

Before we leave this section, I need to make a very important point. The unit in a measurement is just as important as the number itself. After all, if you report the length of the pencil as 7.41, that doesn't mean anything, because it doesn't indicate what unit was used. The pencil is 7.41 inches long, not 7.41 centimeters long. As a result, you must always remember to list the unit you are using with your answer. If you don't list the unit, your answer is wrong!

The unit of a measurement is just as important as the number. You must always list the unit!

## Significant Figures

Believe it or not, we aren't done with measuring length. Once again, measurement is very important in chemistry, so you have to be sure you are doing it correctly. Suppose you were measuring the length of a different pencil, as shown below:


How would you report that length? As far as I can tell, the pencil lines up perfectly with the $19-\mathrm{cm}$ mark. Since the pencil starts at 1 cm and ends at 19 cm , you might be tempted to report the length as 18 cm , but that would be wrong! Remember, a ruler like this is precise enough to be reported to the hundredths of a centimeter. Thus, the correct answer for the length of the pencil is 18.00 cm .

Even though the mathematical value of 18 is the same as 18.00 , from a scientific point of view, there is a huge difference between a measurement of 18 cm and a measurement of 18.00 cm . When a scientist reports a measurement, he must be careful to report only the numbers he has measured. He cannot report numbers that he hasn't measured. If a scientist says, "The length of the pencil is 18 cm ," that means he knows the pencil is somewhere between 17.5 and 18.4 cm long. Since any number between 17.5 and 18.4 can be rounded to 18 , any of those lengths would be consistent with what the scientist said.

However, if the scientist says, "The length of the pencil is 18.0 cm ," that means he knows the pencil's length to a higher precision. He knows that it is somewhere between 17.95 and 18.04 cm long, because any of those measurements round to a length consistent with what he said. If he says, "The length of the pencil is 18.00 cm ," then he knows the length even more precisely. It is somewhere between 17.995 and 18.004 cm long. When a scientist reports a measurement, then, he reports all the digits be measured and no more. In our case, the ruler allows us to read to the hundredths of a centimeter, so we can actually measure the length of the pencil to the hundredths of a centimeter. Regardless of the actual value of the length, then, you must report it to the hundredths of a centimeter, because that tells everyone who reads the measurement how precisely the pencil was measured. When a digit in a number is actually measured, it contributes to the precision of the measurement, and we call it a significant figure.

Significant figure - A digit in a measurement that contributes to the measurement's precision
It's important to remember that the word "significant" doesn't mean "mathematically important." From a mathematical point of view, the two zeroes in 18.00 cm aren't important. However, from a scientific point of view, they are significant, because they have been measured. Scientists must be very careful to report all of their measurements using the proper significant figures.

When you read a measurement, how can you tell whether or not a figure is significant? Here are the rules:

1. All non-zero figures $(1,2,3,4,5,6,7,8$, and 9$)$ are significant.
2. A zero is significant if it is between two significant figures.
3. A zero is also significant if it's at the end of the number and to the right of the decimal point.

When you read a measurement like 1.679 inches, then, all of the figures in that number were measured, because according to rule $\# 1$, they are all significant. In a measurement like 0.00456 cm , however, only the 4,5 , and 6 were measured, because they are significant. Those first three zeroes are not significant
because none of them are between significant figures, and none of them are at the end of the number. See if you can understand how to use these rules in the following examples.

## Example 1.1

## How many significant figures are in 102.0 inches?

Rule \#1 tells us that the " 1 " and " 2 " are significant. Rule \#2 tells us that the zero in between the " 1 " and " 2 " is also significant. The last zero is both at the end of the number and right of the decimal, so Rule \#3 tells us it is significant. In the end, then, all four of the figures in 102.0 inches are significant.

## How many significant figures are in 0.0405 cm ?

The first two zeroes are not between significant figures. They are also not at the end of the number. Thus, they are not significant. Rule $\# 1$ tells us the " 4 " and " 5 " are significant, and that makes the " 0 " in between them significant, according to Rule $\# 2$. As a result, there are three significant figures.

## How many significant figures are in 12,000 inches?

Rule \#1 tells us that the " 1 " and " 2 " are significant. The zeroes are all at the end of the number, but that's not enough. Rule \#3 says they must be at the end of the number and to the right of the decimal. Thus, even though they are mathematically important, they are not significant. That means there are two significant figures.

Don't think of this as some useless exercise about following rules. It is very important to know which figures in a measurement are significant and which are not, because that tells you what was actually measured. Make sure you understand this by answering the following question.

## Comprehension Check

2. How many significant figures are in each of the numbers below?
a. 0.01020 in
b. $12,007 \mathrm{~cm}$
c. $609,000 \mathrm{~cm}$
d. 89.10400 in

The concept of significant figures allows me to make an important distinction. People often talk about precision and accuracy as if they are the same thing, but to a scientist, they are completely different. Precision is determined by the instrument you are using to make your measurement. If you have a very fine measuring instrument, you can report your measurement with a lot of decimal places in it. Accuracy, on the other hand, indicates how correct your measurement is. You can use a very precise instrument, but if you use it wrongly, you will end up with an answer that is precise but not accurate.

For example, we know that a U.S. dollar bill has a length of 15.6 cm . Suppose someone uses a very precise measuring device wrongly and measures the length of a dollar bill to be 12.678 cm . That's a very precise answer, because it has a lot of decimal places in it, but it isn't anywhere close to the correct answer, so it is not accurate. Suppose someone else uses an imprecise instrument and reports that the dollar bill is 16 cm long. Even though the measurement is not very precise (it doesn't even report the answer to tenths of a centimeter), it is very accurate, because the correct length of a dollar bill, reported to two significant figures, is 16 cm . Accuracy and precision, then, are different things. Precision is based on the significant figures involved, while accuracy depends on how close the measurement is to being correct.

## Using Significant Figures in Mathematical Problems

One of the things you will find out about chemistry is that it uses a lot of math! Chemistry is a very mathematical subject, and oftentimes, you will be using measurements in mathematical equations. In fact, the very next section contains an experiment where you must divide one measurement by another measurement. When you use measurements in mathematical equations, you need to be careful to write your answers with the proper number of significant figures. After all, the significant figures tell you what was measured, so it lets you know the precision of a measurement. Once you do math on a measurement, you still have to report your answer with the correct precision.

Suppose, for example, you have two pieces of wood. The first was measured to be 12.12 cm long, and the second was measured by someone else (using a different ruler) to be 6.5 cm long. You glue those pieces of wood end-to-end and want to report the new length without bothering to actually measure it. What would you do? You would just add the two lengths together, right? So the new length would be:

$$
12.12 \mathrm{~cm}+6.5 \mathrm{~cm}=18.62 \mathrm{~cm}
$$

That's close, but it's not quite right. The problem is that the first measurement $(12.12 \mathrm{~cm})$ is more precise than the second measurement $(6.5 \mathrm{~cm})$. When you add them, you are taking a more precise measurement and adding a less precise measurement. What do you think that does to the overall precision?

Think about it this way. Suppose you are a very neat and tidy person. I am not, but let's suppose you are. You keep your room really nice and tidy. Now suppose you have a friend over and you both end up spending time in your room. You read, snack, hang out, etc. If your friend isn't very neat and tidy, what do you think your room will look like when he leaves? It probably won't be nearly as neat and tidy as it usually is. By adding a messy person to your room for a while, your room became a lot less neat and tidy. It's the same with measurements. When you add a less-precise measurement (a messy one) to a more-precise measurement (a neat and tidy one), it ruins the precision (tidiness) of the more-precise measurement. As a result, you can't use that precision anymore. This brings us to a very important point:

When adding and subtracting measurements, you must report your answer to the same precision as the least precise number in the problem.

Once you glue the two pieces of wood together, then, the new length is actually:

$$
12.12 \mathrm{~cm}+6.5 \mathrm{~cm}=18.6 \mathrm{~cm}
$$

Since the measurement of 6.5 has the lowest precision, it determines the precision of the answer. Let's see how this pans out in an example.

## Example 1.2

What is the proper answer when a measurement of 15.423 cm is subtracted from 102 cm ?
First, we can just subtract the two measurements:

$$
102 \mathrm{~cm}-15.423 \mathrm{~cm}=86.577 \mathrm{~cm}
$$

However, because 102 cm has its last significant figure in the ones place, it is less precise than 15.423, which has its last significant figure in the thousandths place. As a result, 102 cm limits the precision of our
answer to the ones place, so the proper answer is 87 cm . That has the same precision as 102 , because they each have their last significant figure in the ones place. Also, note that we had to round up, because we dropped a " 5 ."

There will be times when you multiply and divide measurements. Indeed, in the very next section, you will be dividing measurements. You will also have to be careful to report your answers to the proper precision as well, but because multiplication and division are fundamentally different from addition and subtraction, the rule for significant figures is different.

When multiplying and dividing measurements, you must report your answer with the same number of significant figures as the measurement which has the fewest significant figures.

In other words, you count the number of significant figures in each measurement. Whichever has the fewest significant figures limits your answer. You report your answer to that number of significant figures. Here are two examples of how that works:

## Example 1.3

## What is the proper answer when a measurement of 15.423 cm is divided by 102 cm ?

First, we can just divide the two measurements:

$$
15.423 \mathrm{~cm} \div 102 \mathrm{~cm}=0.151205882353
$$

Depending on your calculator (yes, you should use a calculator in this course), your answer might have even more digits in it. That's fine. The answer above already has far too many! The measurement of 15.423 cm has five significant figures, while the measurement 102 cm has three. Since the lowest number of significant figures is three, the answer can have only three. The answer, therefore, is $\underline{0.151}$. Remember, that first zero is not significant, as it is not between significant figures or at the end of the number.

Before we get to the next problem, I want to point out what happened to the unit (cm). Each measurement has a unit. If you don't write your unit with your measurement, it is wrong. When I do math with measurements, I include those units in the math, just as if they were numbers. So in the equation above, I am dividing 102 by 15.423 , but I am also dividing cm by cm . But what happens when you divide something by itself? It becomes 1, right? For a unit, that means the unit cancels out! So there is no unit in the answer, because the only units in the problem (cm) cancelled themselves out when one was divided by the other.

## What is the proper answer when a measurement of 7.0 in is multiplied by 4.209 in ?

First, we just multiply the two measurements:

$$
7.0 \text { in } \cdot 4.209 \text { in }=29.463 \mathrm{in}^{2}
$$

Remember from algebra that a dot between two numbers indicates multiplication. So the two numbers and the two units get multiplied together. What happens when you multiply a number by itself? You get the square of the number. Remember from algebra that $x \cdot x=x^{2}$. In the same way, in $\cdot$ in is in ${ }^{2}$. Of course, we also have to consider significant figures. The first measurement has two significant figures, while the second has four. Thus the answer can have only two, so the proper answer is $\underline{29} \mathrm{in}^{2}$.

## Comprehension Check

3. What is the correct answer when 3.1 in are added to 8.991 inches?
4. What is the correct answer when a measurement of 45.5 cm is multiplied by 9 cm ?

## A Relationship Between Units

Now that you understand significant figures and how to use them in mathematical equations, let's go back to measuring length. We have measured it in both inches and centimeters. Since both of those units measure length, there ought to be a relationship between them. After all, in the English system, you can measure length in inches, feet, yards, and miles. There is a relationship between each of these units: 12 inches in a foot, 3 feet in a yard, and 1,760 yards in a mile. Centimeters comes from a completely different unit system (the metric system), but since they also measure length, there should be some relationship between centimeters and inches. In the following experiment, you are going to figure out that relationship.

## Experiment 1.1: Determining the Relationship Between Inches and Centimeters

## Materials

- A softcover book
- A ruler that uses both centimeters and inches (or two different rulers, one that uses centimeters and one that uses inches)


## Instructions

1. Measure the width of the book in centimeters. Write it down as a decimal to the proper number of significant figures. Remember, if the ruler has marks for tenths of a centimeter, you can report your answer to hundredths of a cm .
2. Measure the width of the book in inches. Write it down to the proper number of significant figures. Remember, you can report your answer to hundredths of an inch, as long as the ruler is marked off in $16^{\text {th }}$ s of an inch.
3. Divide the answer you got in step 1 by the answer you got in step 2 . Write it down to the proper number of significant figures. This means you must count the number of significant figures in each measurement you made, and your answer needs to have the same number of significant figures as the measurement which had the least. (There are sample calculations for the experiment after the answers to the "Comprehension Check" questions.)
4. Repeat steps 1-3, but this time measure the length of the book.
5. Repeat steps 1-3 one more time, but this time measure the width of the book's spine to get the thickness of the book.
6. Put the book and rulers away.

Before we look at the numbers you got in your experiment, let's look at the units. When you divide centimeters by inches, what do you get? The units don't cancel, like when you are dividing inches by inches. There isn't really anything they do. You just have $\mathrm{cm} \div$ in. Usually, we write that as a fraction: cm/in.

Believe it or not, that actually means something. It is the ratio between centimeters and inches. Usually, we read a fraction like that as, "centimeters per inch." In other words, the number that goes with that fraction actually tells us how many centimeters are in an inch! Look at the numbers you got. Ideally, they should all be the same, because in each case, you took the same measurement in centimeters and
inches and divided the two. This should give you the number of centimeters in an inch. If you did your measurements correctly, all three of your numbers should be consistent with a value of $2.54 \mathrm{~cm} / \mathrm{in}$, which is the accepted value for how many centimeters are in an inch.

What does it mean to be "consistent with?" You have to think about significant figures. This is why they are so important. When you made your measurement, you estimated the last significant figure, didn't you? After all, the metric ruler was marked off in tenths of a centimeter. You estimated where the edge of the book fell in between the marks to report your answer to the hundredths place. This tells us something pretty important:

## There is always some error in the last significant figure of a measurement.

When scientists compare measurements, they take this into account. For example, suppose you got an answer of 2.52 centimeters per inch. That's consistent with the correct value. After all, there is error in the last significant figure. Because we know that, two measurements can be different in their last significant figure and still be consistent with one another. In the same way, if you got an answer of 2.57 centimeters per inch, that's still consistent with 2.54 centimeters per inch.

How much error is acceptable in the last significant figure of a measurement? Would 2.59 centimeters per inch be consistent with 2.54 centimeters per inch? In some ways, it's a judgment call that varies from scientist to scientist. For the purposes of this course, as long as one measurement's last significant figure is no more than 3 different from the last significant figure of the other measurement, the two are consistent with one another.

## Converting Between Units

Now that you know a relationship between centimeters and inches, you can switch between the two rather easily. This process is called unit conversion, and you will use it over and over again in chemistry. There are several ways you can do a unit conversion, but I am going to teach you a method that applies to many different situations in chemistry. As a result, even if you are already comfortable converting inches into centimeters and vice-versa, you still need to learn this method, as it will apply to a lot of what you will study in this course.

The method you will use is called the factor-label method, and it makes use of a fact that you learned when you did arithmetic. When multiplying fractions, you were probably taught to look for numbers that appear in both the numerator and denominator of the fractions you were multiplying. For example, in the following multiplication:

$$
\frac{17}{25} \times \frac{25}{21}
$$

You should have been taught that the 25 's cancel, because in a fraction, you are simply dividing the top number by the bottom number. Since $25 \div 25=1$, the only numbers left are the 17 and 21 . Thus:

$$
\frac{17}{25} \times \frac{25}{21}=\frac{17}{21}
$$

While you are probably used to recognizing this fact when it comes to numbers, remember that the same thing applies to units! After all, what happened in Example 1.3 when we divided centimeters by centimeters? They cancelled, leaving no unit!

How does this help? Well, let's go back to the results of your experiment. You learned that there are 2.54 centimeters per inch. Even if you didn't get that exact answer, it's the accepted value, so it's the one we are going to use. What does that mean? It means that a measurement of 2.54 centimeters is the same as a measurement of one inch. In other words:

$$
2.54 \mathrm{~cm}=1 \mathrm{in}
$$

This is called a conversion relationship, because it tells you the relationship between two different units.
Since the two measurements in a conversion relationship are equal, think about what the following fractions represent:

$$
\frac{2.54 \mathrm{~cm}}{1 \mathrm{in}} \text { OR } \frac{1 \mathrm{in}}{2.54 \mathrm{~cm}}
$$

When I take two equal things and put them over each other in a fraction, what happens? They cancel each other out, equaling 1. What does that tell you? It tells you that you can multiply anything by either of the fractions above, and you aren't changing its value, because you are just multiplying by something that equals 1 , and multiplying by 1 doesn't change the value of anything.

So, let's suppose I have a measurement like 12.0 inches. How many centimeters would that be? Well, I can take that measurement and make it into a fraction by putting it over 1. After all, you can make any number a fraction by just putting it over 1 :

$$
\frac{12.0 \mathrm{in}}{1}
$$

Now I can multiply this by either of the fractions I showed you before, because they both equal 1. However, I will be very careful about how I do that. I will multiply by the fraction on the left. Why? Look what happens:

$$
\frac{12.0}{1} \times \frac{2.54 \mathrm{~cm}}{1 \mathrm{in}}=30.5 \mathrm{~cm}
$$

Notice that the unit of inches appears in the numerator of the first fraction and the denominator of the second. Because of this, it cancels. That leaves us with only one unit - centimeters. That's the unit we wanted! So we now know that 12.0 inches is the same as 30.5 cm .

This is the way you will use the factor-label method when converting units. You will put the measurement you want to convert over 1 so it becomes a fraction. Then, you will multiply by a fraction you build from the conversion relationship. The fraction you use should have the unit you want to get rid of on the bottom and the unit you want to keep on the top. That way, the old unit cancels, and the new unit is what's left.

I will show you a couple of examples in a moment, but first, let's worry about the significant figures. Most conversion relationships don't bother to list all the significant figures. This is because most of the conversion relationships you will see, including the one between inches and centimeters, are exact. That means $1.00000000000000000 \ldots$ inch is exactly $2.54000000000000000 \ldots$ centimeters. In other words, both numbers in the conversion relationship have an infinite number of significant figures. This means when you do the conversions in this chapter, you don't have to worry about the significant figures in the conversion relationship. You only have to count the significant figures in the original measurement
and make sure your answer has the same number of significant figures. In the conversion I did above, 12.0 has three significant figures, so the answer had to be written to three significant figures, which is why I rounded it to 30.5 cm . Let's see how this works in a couple of example problems.

## Example 1.4

## How many inches are in 20.12 cm ?

We start by taking the measurement and putting it over 1 :


Now we multiply by a fraction made from the conversion relationship ( $2.54 \mathrm{~cm}=1 \mathrm{in}$ ), but the fraction must have centimeters on the bottom so it cancels:

$$
\frac{20.12 \mathrm{em}}{1} \times \frac{1 \mathrm{in}}{2.54 \mathrm{em}}=7.921 \mathrm{in}
$$

Remember, the only thing we have to worry about is the number of significant figures in the original measurement. It has four, so the answer must have four. That's why the answer is $\underline{7.921 \text { in. }}$

## An Apple iPhone 6+ has a length of 6.22 inches. What is the length in cm ?

We start by taking the measurement and putting it over 1 :

$$
\frac{6.22 \mathrm{in}}{1}
$$

Now we multiply by a fraction made from the conversion relationship ( $2.54 \mathrm{~cm}=1 \mathrm{in}$ ), but the fraction must have inches on the bottom so it cancels:

$$
\frac{6.22 \mathrm{im}}{1} \times \frac{2.54 \mathrm{~cm}}{1 \mathrm{im}}=15.8 \mathrm{~cm}
$$

Since there are three significant figures in the original measurement, there must be three in the answer, which is 15.8 cm .

## Comprehension Check

5. A laptop computer is 33.56 cm wide. How many inches wide is the laptop?

## Prefixes in the Metric System

Throughout my discussion about how to measure distance, I have talked about only two units: inches and centimeters. Obviously, there are a lot more units. In the English system, you can also measure length in feet, yards, and miles. If I wanted to measure the width of a room, I would probably use feet. If I wanted to measure the length of something longer, like a football field, I would probably use
yards. If I wanted to measure the distance between cities, I would probably use miles. Each unit is good for measuring a specific kind of distance.

In the metric system, there are also different units, but unlike the English system, all these units are defined relative to a base unit. The base unit for measuring distance in the metric system is the meter (abbreviated as " $m$ "). It is about the same length as a yard, so whatever you might measure in yards in the English system can be easily measured in meters in the metric system. An American football field, for example, is 100.0 yards from goal line to goal line. That's the same as 91.44 meters.

So what's a centimeter? It's one hundredth (0.01) of a meter. In the metric system, you modify the base unit to come up with a unit that is appropriate for what you are measuring. When you measure the length of something like a pencil, using centimeters makes sense, because a pencil is much, much shorter than a meter. So you modify the base unit of the metric system with the prefix centi, which means 0.01.

If you want to measure something even smaller, say the length of a grain of rice, you might want to use an even smaller unit. The millimeter is one thousandth ( 0.001 ) of a meter, because the prefix milli means 0.001 . A long grain of rice is about 7 millimeters long, while a short grain is about 4 millimeters long. On the other hand, if you want to measure something really long, like the distance between two cities, you might use the kilometer, which is 1,000 meters. The driving distance between Washington, DC and New York, NY, for example, is 365 kilometers.

There are a lot more prefixes in the metric system, which makes it very flexible. The table below summarizes a few of them. Please note that you do not need to know the meaning of all these prefixes. The ones I expect you to know by heart (kilo, centi, and milli) are put in boldface type. The others are there just to give you a feel of how flexible the metric system is.

## Metric Prefixes and Their Meanings

| Prefix | Abbreviation | Meaning |  | Prefix | Abbreviation | Meaning |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| mega | M | $1,000,000$ |  | centi | c | $\mathbf{0 . 0 1}$ |
| kilo | k | $\mathbf{1 , 0 0 0}$ |  | milli | m | $\mathbf{0 . 0 0 1}$ |
| hecto | H | 100 | micro | $\boldsymbol{\mu}$ | 0.000001 |  |
| deca | Da | 10 | nano | n | 0.000000001 |  |

When you are measuring distance, then, you can choose a prefix to put in front of the base unit. The prefix you use will determine the size of the unit you are using.

Now, of course, because we know the meanings of these prefixes, it is rather easy to convert between one unit and another in the metric system. We can use the factor-label method just as we did before. See how this works in an example.

## Example 1.5

## An Olympic pool is 0.05 kilometers in length. How long is that in meters?

In order to do a conversion, we need a conversion relationship. The prefix "kilo" means " 1,000 ," so we just write " $1 \mathrm{~km}=$ " and then replace the prefix with its meaning:

$$
1 \mathrm{~km}=1,000 \mathrm{~m}
$$

Notice how the left-hand side of this conversion relationship has the unit with the prefix (km), while the right-hand side has the prefix's meaning $(1,000)$ followed by the base unit (m). Now we just put the original measurement over 1 to make it a fraction and multiply by a fraction made from the conversion relationship, making sure km cancels:

$$
\frac{0.05 \mathrm{~m}}{1} \times \frac{1,000 \mathrm{~m}}{1 \mathrm{~m}}=50 \mathrm{~m}
$$

There is only one significant figure in 0.05 , so there can be only one significant figure in the answer, which is $\underline{50 \mathrm{~m}}$.

## Comprehension Check

6. The diameter of a U.S. dime is 17.9 millimeters. How many meters is that?

## Scientific Notation

Before we move on and start learning about measuring area and volume, I want to bring up an important point related to significant figures. Sometimes, we have to change the way we write down numbers in order to make sure the significant figures are communicated properly. To see what I mean, consider the Olympic swimming pool mentioned in Example 1.5. Suppose you measure it with a measuring tape marked off in meters. You can estimate between the marks, but the pool's edge falls right on the 50 -meter mark. That means you would write its length as 50.0 meters. How many significant figures does that measurement have? It has three.

Now, suppose I asked you to convert that into centimeters. What would you do? Well, "centi" means 0.01 , so you would write down the conversion relationship, $1 \mathrm{~cm}=0.01 \mathrm{~m}$. You would then multiply by a fraction made from that conversion relationship so that meters cancel:

$$
\frac{50.0 \mathrm{~m}}{1} \times \frac{1 \mathrm{~cm}}{0.01 \mathrm{~m}}=5,000 \mathrm{~cm}
$$

There is a problem with that answer. Can you see what it is? The original measurement has three significant figures, so the answer needs to have three. How many significant figures are in $5,000 \mathrm{~cm}$ ? Only one! The zeroes are at the end of the number, but they are not to the right of the decimal place. Thus, they are not significant. In order to make two of them significant so that the entire number can have three significant figures, I need to get two of the zeroes to be right of the decimal. How can I do that?

The answer is to use scientific notation. When you write a number in scientific notation, there is only one figure to the left of the decimal. All other figures are written to the right of the decimal. To keep that from changing the actual value of the number, you then multiply the number by 10 raised to some exponent. The proper answer to the equation above, for example, is $5.00 \times 10^{3} \mathrm{~cm}$. Notice that there are now three significant figures in the measurement, because the two zeroes are at the end of the number and to the right of the decimal. In addition, the value is the same, because $10^{3}$ is 1,000 , and $5.00 \times 1,000=$ 5,000.

So, when you have a large number, you can change it into scientific notation by moving the decimal place to the left until there is only one figure to the left of it. Then, you multiply by 10 raised to the same power as the number of places you moved the decimal. Converting a number like $14,000,000$ to scientific notation, then, looks like this:

## $14000000 \underset{\text { sulupp }}{ } 1.4 \times 10^{7}$ <br> move the decimal to

 the left seven places.the result
by $10^{7}$.

If you need some of the zeroes to be significant in order to properly follow the rules of significant figures, you can just add as many as you need after the " 4 ," because any zero at the end of the number and to the right of the decimal is significant.

Now, of course, you can go the other way as well. Suppose you have a really small number, like 0.00000014 . You might find it annoying to write all those zeroes. Since they aren't significant, we can get rid of them with scientific notation. In this case, however, you have to move the decimal place to the right to get the " 1 " to the left of it, so you have to multiply by 10 raised to a negative exponent:

## $0.00000014 \Longrightarrow 1.4 \times 10^{-7}$ <br> move the decimal to the right seven places. the result by $10^{-7}$.

Of course, if you have a number in scientific notation, you can always convert it back to standard decimal notation by doing the reverse. If the number is multiplied by 10 to a positive exponent, it's a large number, so you move the decimal to the right by the number of places given in the exponent. If the number is multiplied by 10 to a negative exponent, you know the number is small, so you move the decimal to the left by the number of places given in the exponent. If this seems a bit confusing, the following examples should clear things up.

## Example 1.6

## Convert 610,000 into scientific notation so that it has four significant figures.

To get it into scientific notation, we need to move the decimal place to in between the " 6 " and the " 1 ." That means moving it five places. Since the number is big, the exponent on the ten will be positive 5 . To have four significant figures, we will need to keep two of the zeroes. That means the answer is $\underline{6.100 \times 10^{5}}$.

## Convert 0.00000456 into scientific notation.

To get it into scientific notation, we need to move the decimal place to in between the " 4 " and the " 5 ." That means moving it six places. Since the number is small, the exponent on the ten will be -6 . That means the answer is $4.56 \times 10^{-6}$.

Convert $8.120 \times 10^{-12}$ into decimal notation.
In this case, we need to get out of scientific notation. Since the exponent is negative, we know the number is small. That means moving the decimal place to the left a total of 12 places (the value of the exponent). Thus, the answer is 0.000000000008120 .

## Convert $3.4 \times 10^{9}$ into decimal notation.

In this case, we need to get out of scientific notation. Since the exponent is positive, we know the number is large. That means moving the decimal place to the right a total of 9 places (the value of the exponent). Thus, the answer is $3,400,000,000$.

## Comprehension Check

7. Convert the following numbers to scientific notation so they each have five significant figures:
a. $53,000,000,000$
b. 0.0094230
8. Convert the following numbers into decimal notation:

$$
\begin{array}{ll}
\text { a. } 8.612 \times 10^{-8} & \text { b. } 6.965 \times 10^{4}
\end{array}
$$

Now that you understand scientific notation, you might be wondering when you should use it. In general, whether or not you use scientific notation is up to you. A number like 5,467 is identical in every way to $5.467 \times 10^{3}$, so you can report it using either decimal or scientific notation. The only time you are required to use scientific notation is when the significant figures can only be properly expressed with scientific notation.

For example, suppose you do some math on a few measurements and end up with an answer of 700. However, suppose the rules of significant figures require you to report that answer to two significant figures. There is no way to do it with decimal notation. The only way to properly report your answer is to use scientific notation so that one of the zeroes can be made significant. In this case, then, you would have to use scientific notation and report your answer as $7.0 \times 10^{2}$.

## Measuring Area and Volume: Derived Units

Now that you know the ins and outs of measuring distance, it's time to learn about a couple of measurements that can result from measuring distance and then doing some math. Suppose, for example, you wanted to put carpet in a room. One of the first things you will probably do is go shop for the kind of carpet you want. When you find the right carpet, the price you are given will be based on square feet, square yards, or square meters. What does that mean?

A squared distance unit is a measurement of area. If the room that you are carpeting has a length of 3.00 meters and a width of 3.50 meters, you can calculate the area of the room by multiplying the length times the width. When you multiply the numbers, you get 10.5, and when you multiply the units, you have meters-meters, or meters ${ }^{2}$ (abbreviated as " $\mathrm{m}^{2 "}$ ). That unit is called "square meters," which is a metric unit for area. Units like this are called derived units, because you get them by doing math with units.

While area is an important thing to measure when you are laying carpet or trying to get a feel for how much room is in a house, a more important thing for chemists to measure is volume. After all, matter is defined as something that has mass and takes up space. If you measure an object's volume, you are measuring how much space it takes up. Consider, for example, a box. How would you determine how much space it takes up? If you measure its length, width, and height and then multiply the three together, you will have measured the volume of the box, because volume $=$ length $\times$ width $\times$ height. What unit will you get when you do that? Well, let's suppose you measured the length to be 1.20 m , the width to be
0.450 m , and the height to be 1.80 meters. When you multiply the numbers, you get 0.972 , and when you multiply the units, you get $\mathrm{m}^{3}$. The volume of the box is $0.972 \mathrm{~m}^{3}$. The unit $\mathrm{m}^{3}$ is called "cubic meters," and it is an example of a volume unit. Any distance unit (like inches, centimeters, feet, etc.) cubed is a measure of volume.

Since a meter is pretty big, volumes that are measured in cubic meters are pretty big as well. Chemists generally work with smaller volumes, so a common volume unit chemists use is $\mathrm{cm}^{3}$, or cubic centimeters. To get that volume unit, you would just measure the length, width, and height in centimeters. When you multiplied them together, you would get $\mathrm{cm}^{3}$. For example, many sugar cubes have a length of 1.27 cm , a width of 1.27 cm , and a height of 1.27 cm . When you multiply those together, you get a volume of $2.05 \mathrm{~cm}^{3}$.

While any distance unit cubed is a valid unit for volume, that's not
 the only unit in which volume is measured. When you buy a bottle of water at the store, for example, how is the volume of the bottle reported? It's generally reported in liters. It might be reported in quarts or gallons, which are English units for volume, but in many cases, it is reported in liters. That's another common metric unit for volume, and chemists use it quite a lot. Once again, however, a liter is a fairly large volume. Chemists generally use smaller volumes than that, so the most common unit you will find for volume in chemistry is the milliliter (abbreviated as " mL "). Now remember what you have learned about the metric system. The prefix "milli" is one you need to have memorized. It means 0.001 . So, a milliliter is 0.001 liters.

How do we measure volume in liters or milliliters? Chemists often use a graduated cylinder, which is illustrated on the left. This device is a glass (or plastic) cylinder into which a liquid can be poured. It has markings (sometimes called "graduations") that allow you to read the volume of the liquid that has been poured into it. However, because the liquid poured into the graduated cylinder interacts with the glass (or plastic), the liquid's surface isn't flat. Instead, it is curved. Usually, it is curved downwards, as shown in the picture on the left. This curve is called a meniscus (muh nis' kus), and to read the volume, you always look at the bottom of the meniscus. When reading any scale like this, it is also important to look at the scale straight on. Look at the scale with your eyes on the same level as the lines you are reading.

The numbers on the graduated cylinder shown on the left represent milliliters. Since there are 10 lines between each marking of 10 mL , the small lines on the cylinder mark off volumes of 1 mL each. You can estimate the volume between the lines, so you can report the answer to tenths of a milliliter. Look, for example, at the bottom of the meniscus in the drawing. The line it is above represents 87 mL , because it is the seventh line above 80 mL . However, it is clearly above that mark. At the same time, it is not directly in between the 87 mL mark and the next mark, which represents 88 mL . It is much closer to the 87 mL mark than the 88 mL mark, so it is a bit more than 87 mL but a lot less than 88 mL . I would say it is about 87.1 mL . You might call it 87.2 mL or 87.3 mL . That's fine. All of those answers would be valid, because they are all consistent within the precision of the graduated cylinder.

Since any cubed distance unit (such as $\mathrm{m}^{3}, \mathrm{~cm}^{3}$, or feet ${ }^{3}$ ) is a valid volume unit, and since the liter (or any prefix version of liter, such as mL ) is also a valid volume unit, there should be a relationship between them. In the following experiment, you are going to figure out what that relationship is.

## Experiment 1.2: Determining the Relationship Between Cubic Centimeters and Milliliters

## Materials

- A funnel
- A $50-\mathrm{mL}$ graduated cylinder
- A medicine dropper
- A small cylindrical container, like a pill bottle or the jars in which spices are sold (It needs to be able to hold water.)
- A metric ruler
- Water and a sink
- Safety goggles (Wear them throughout the experiment.)


## Instructions

1. Measure the height of the container in cm , reporting your answer to hundredths of a cm .
2. Measure the diameter of the bottom of the cylindrical container. Do this by positioning the ruler so that the edge with the marks passes through the center of circle that forms the opening of the container. Measure the distance from one edge of the opening to the other, and that's the diameter. Once again, report your answer to hundredths of a cm .
3. Divide the diameter by 2 to get the radius. Since 2 is exact, keep the same number of significant figures as you had in the measurement of the diameter.
4. The volume of a cylinder is given by $\pi r^{2} h$, where " $r$ " is the radius, and " $h$ " is the height. Use that formula to calculate the volume of the cylindrical container. Remember to report your answer to the proper number of significant figures. Since you are squaring a cm unit and then multiplying by another cm unit, your unit is $\mathrm{cm}^{3}$, which is what you would expect for a volume measurement.
5. Fill the container you just measured with water. Fill it so it is completely full. Allow some water to spill out to make sure it is completely full.
6. Put the funnel in the top of the graduated cylinder.
7. Slowly and carefully pour water from the container into the graduated cylinder, using the funnel so that no water spills.
8. If you can transfer all of the water from the container to the graduated cylinder without reaching the $50-\mathrm{mL}$ mark, read the volume and report it to tenths of a mL . Remember, your eyes need to be level with the mark you are reading to get the correct volume. Skip to step 14.
9. If the water level gets very near the $50-\mathrm{mL}$ mark, stop pouring. If it accidentally goes above the $50-\mathrm{mL}$ mark, remove the funnel and pour water from the graduated cylinder back into the container.
10. Once the water is near but not touching the $50.0-\mathrm{mL}$ mark, take out the funnel and use the medicine dropper to slowly transfer water from the container to the graduated cylinder until the bottom of the meniscus is right at the $50-\mathrm{mL}$ mark. That's 50.0 mL of water.
11. Squirt all the water left in the medicine dropper back into the container.
12. Dump the water out of the graduated cylinder and into the sink.
13. Repeat steps 6-11 until you have transferred all the water from the container to the graduated cylinder.
14. Take the last volume you read (the one in step 8 ) and add 50.0 mL to it for each full graduated cylinder you dumped out. The result will be the volume of water that the container held in mL .
15. Compare the volume in cubic centimeters (you calculated that in step 4) to the volume in milliliters, which you just figured out. Are they close to one another? They should be.
16. Clean up your mess.

If you did everything perfectly in the experiment, the two volumes should have been the same. That's because one cubic centimeter is the same as a milliliter:

$$
1 \mathrm{~cm}^{3}=1 \mathrm{~mL}
$$

Most likely, your two volumes weren't identical, but they were probably close. Some of the water probably ended up spilling or sticking to the sides of the graduated cylinder, container, and medicine dropper, which would cause some error in your results. We call that experimental error, and it is inevitable. All experiments have error in them. Good experiments, however, keep the error to a minimum.

## Converting Between Volume Units

Since you now know that milliliters and cubic centimeters are the same, it is fairly easy to convert between different volume units. Consider the following example:

## Example 1.7

The volume of a box was measured to be $3,245 \mathrm{~cm}^{3}$. What is its volume in liters?
We know that $\mathrm{cm}^{3}$ and mL are the same. That means the volume of the box is also $3,245 \mathrm{~mL}$. Since "milli" means " 0.001 ," we know the conversion relationship between milliliters and liters is:

$$
1 \mathrm{~mL}=0.001 \mathrm{~L}
$$

Now we just put the original measurement over 1 to make it a fraction, and we multiply by the conversion relationship so that milliliters cancel:

$$
\frac{3,245 \mathrm{~mL}}{1} \times \frac{0.001 \mathrm{~L}}{1 \mathrm{mI}}=3.245 \mathrm{~L}
$$

The volume is 3.245 L .
That was pretty easy, but these conversions can get a lot trickier. For example, suppose someone measured the volume of a box in $\mathrm{m}^{3}$. That's obviously a volume unit, because it is a distance unit cubed. With just what you know now, could you convert the volume into liters? You could, because you know that $\mathrm{cm}^{3}$ and mL are the same. Thus, you would just have to convert $\mathrm{m}^{3}$ into $\mathrm{cm}^{3}$. That would also be the volume in mL . Then, you could convert from mL to liters. However, that first step is trickier than you might think. Study the following example to see what I mean.

## Example 1.8

## The volume of a certain refrigerator is $2.88 \mathrm{~m}^{3}$. How many liters is that?

Since we know how $\mathrm{cm}^{3}$ relates to mL , we first need to convert from $\mathrm{m}^{3}$ to $\mathrm{cm}^{3}$. You might think that's easy. After all, you just put the original measurement over 1 to make it a fraction, and then multiply by a fraction that contains the conversion relationship $(1 \mathrm{~cm}=0.01 \mathrm{~m})$ and cancels out the $\mathrm{m}^{3}$ :

$$
\frac{2.88 \mathrm{~m}^{3}}{1} \times \frac{1 \mathrm{~cm}}{0.01 \mathrm{~m}}
$$

But that's not correct. In order for the conversion to work, you must completely cancel the $\mathrm{m}^{3}$ unit. Right now, $\mathrm{m}^{3}$ doesn't cancel. In order to cancel the $\mathrm{m}^{3}$ on the top of the first fraction, I need $\mathrm{m}^{3}$ on the bottom of the second fraction. I don't have $\mathrm{m}^{3}$. I only have m . How can I fix this? I can cube the second fraction:

$$
\frac{2.88 \mathrm{~m}^{3}}{1} \times\left(\frac{1 \mathrm{~cm}}{0.01 \mathrm{~m}}\right)^{3}
$$

When you cube a fraction, you cube both the top and the bottom of the fraction. So I have to cube 1 cm , and I have to cube 0.01 m . When I cube 1 , I just get 1 . When I cube $\mathrm{cm}, \mathrm{I}$ get $\mathrm{cm}^{3}$. When I cube $0.01, \mathrm{I}$ get 0.000001 . When $I$ cube $\mathrm{m}, \mathrm{I}$ get $\mathrm{m}^{3}$. So now the equation becomes:

$$
\frac{2.88 \mathrm{~m}^{3}}{1} \times \frac{1 \mathrm{~cm}^{3}}{0.000001 \mathrm{~m}^{3}}
$$

Now look what happens. There is $\mathrm{a}^{3}$ on the top of the first fraction and on the bottom of the second fraction. Therefore, $\mathrm{m}^{3}$ cancels:

$$
\frac{2.88 \mathrm{~m}^{3}}{1} \times \frac{1 \mathrm{~cm}^{3}}{0.000001 \mathrm{~m}^{3}}=2,880,000 \mathrm{~cm}^{3}
$$

Notice also that the unit left over is $\mathrm{cm}^{3}$, which is the unit we wanted. When you need to convert between units that are cubed, then, you need to cube the conversion relationship. In the same way, if we were dealing with area, which is measured in $\mathrm{m}^{2}$, and want to convert to $\mathrm{cm}^{2}$, we would need to square the conversion relationship so that $\mathrm{m}^{2}$ cancels and the unit left is $\mathrm{cm}^{2}$.

Of course, this isn't quite the answer. The question asked for liters, and we have $\mathrm{cm}^{3}$. Of course, $\mathrm{cm}^{3}$ is the same as mL , and it is easy to convert mL to liters:

$$
\frac{2,880,000 \mathrm{~mL}}{1} \times \frac{0.001 \mathrm{~L}}{1 \mathrm{~mL}}=2,880 \mathrm{~L}
$$

$2.88 \mathrm{~m}^{3}$ is the same as 2,880 liters.

## Comprehension Check

9. Convert 7.4 liters into $\mathrm{cm}^{3}$.
10. A box is measured to have a volume of 11.7 liters. What is its volume in $\mathrm{m}^{3}$ ?

## Measuring Mass

Matter takes up space, but it also has mass. You have learned how to measure the space it takes up, so now it's time to learn how to measure mass. You might think that's easy. After all, the more mass something has, the heavier it will be. So if you measure how heavy something is, you are measuring its mass. While that sounds reasonable, it's not really correct. There is a big difference between mass and


This photograph shows Neil Armstrong in his space suit on the moon. While the weight of his body and equipment was 360 pounds on earth, it was only 60 pounds on the moon.
weight. Consider, for example, famous astronaut Neil Armstrong, the first man to walk on the moon. When he trained on earth, the combined weight of his body and his space suit was about 360 pounds. When he walked on the moon, however, the combined weight of his body and his space suit was about 60 pounds.

Why was there such a difference? Did he and his space suit lose a lot of mass between the time he last trained on earth and when he walked on the moon? Of course not! Ignoring changes in Armstrong's diet and exercise, the mass of his body and space suit stayed the same the entire time. However, the combined weight changed because the earth exerts a stronger gravitational force on things than does the moon. As a result, when he got to the moon, the gravitational force he experienced was significantly lower than what he experienced on the earth. This illustrates the very important distinction between mass and weight. Weight is a measure of how strongly an object is pulled by gravity. Mass, on the other hand, is a measure of how much matter is in an object.

Weight - A measure of how strongly gravity pulls on an object
Mass - A measure of how much matter exists in an object
As long as an object doesn't lose matter, its mass stays the same. However, the weight of an object changes depending on the gravity to which it is being exposed.

Let's think a bit more about the difference between weight and mass. When people in the United States measure their weight, they usually do so in pounds, but that's not the only unit used to measure weight. For example, some people in Great Britain and Ireland measure their body weight in stones, a unit that has been in use since at least the middle ages. One stone is the same as 14 pounds. Neither of those units are metric, however. A common metric unit for weight is the Newton, which is named after Sir Isaac Newton, one of the most brilliant scientists who ever lived.

Since mass and weight are different things, they are measured with different units. The base metric unit for mass is the gram (abbreviated as " $g$ "), so whenever you see that unit, you know that mass is being measured, not weight. Neil Armstrong and his space suit, for example, had a mass of 160,000 grams. Whether he was on earth or the moon, that measurement stayed the same, because mass is not dependent on the gravity to which you are exposed. What about the English unit for mass? It happens to be my favorite unit. Believe it or not, the English unit for mass is called the slug. In English units, Neil Armstrong and his space suit had a mass of 11 slugs. Here's a way to visualize the difference in mass between a gram and a slug: A standard-sized, metal paper clip has a mass of about 1 gram, while it would take more than 14,500 of those paper clips to get a mass of one slug.

Now that you know the difference between mass and weight, you might be a bit surprised to learn that most of the time, we measure mass by measuring weight. In the experiment kit that comes with this course, for example, there is a scale that reads grams. That means it is measuring mass. However, the scale works by actually measuring the weight of what you put on it and then converting that weight to mass. This works because there is an equation that relates mass and weight. You don't need to know that equation. You just need to know that it exists, and it depends on the planet you are on and altitude you are at. As long as you stay on the same planet at roughly the same altitude, the equation that allows you to measure weight and then calculate mass stays the same.

If you travel to another planet, of course, the conversion between weight and mass changes. Even if you stay on earth, however, the conversion can change somewhat. For example, a person who weighs 125.0 pounds at sea level on the earth will weigh 124.6 pounds at the top of Mt. Everest, which is about 1.8 kilometers above sea level. That's not much of a change, so your weight isn't strongly dependent on altitude. Nevertheless, the change is measurable. As long as you aren't doing your experiments at the top of Mt. Everest, however, you can rest assured that the mass you read from your scale is accurate, even though the scale actually measures weight.

In chemistry, you will measure most of your masses in grams. Since it is a metric unit, however, you can use all the prefixes found in the table on page 12 with it. As a result, you will find that you have to convert between mass units from time to time. This gives me the opportunity to discuss a more complicated type of conversion in the metric system.

## Example 1.9

## The mass of a can of soup is 0.490 kg . What is its mass in mg ?

What is a mg? Remember that the prefix "milli" is abbreviated with an " $m$," so "mg" means milligram. In the same way, "k" abbreviates "kilo," so "kg" means kilogram. That means this is a conversion between kilograms and milligrams. Wait a minute. How do we do that? We don't have a conversion relationship between kg and mg . However, we do know the conversion relationship between kg and g , and we also know the conversion relationship between g and mg . As a result, this is a two-step conversion. We will convert from kg to g , and then we will convert from g to mg .

To get from kg to grams, we put the original measurement over 1 and multiply by a fraction that includes the conversion relationship ( $1 \mathrm{~kg}=1,000 \mathrm{~g}$ ) and causes the kg to cancel:

$$
\frac{0.490 \mathrm{fg}}{1} \times \frac{1,000 \mathrm{~g}}{1 \mathrm{~kg}}=4.90 \times 10^{2} \mathrm{~g}
$$

Notice that the original measurement has three significant figures, so to keep three significant figures in the answer, I had to use scientific notation. Now that we have grams, we do the same thing again, this time using the conversion relationship between mg and $\mathrm{g}(1 \mathrm{mg}=0.001 \mathrm{~g})$ and making sure that grams cancel:

$$
\frac{4.90 \times 10^{2} \mathrm{~g}}{1} \times \frac{1 \mathrm{mg}}{0.001 \mathrm{~g}}=4.90 \times 10^{5} \mathrm{mg}
$$

So, a mass of 0.490 kg is the same as $4.90 \times 10^{5} \mathrm{mg}$. This is the way you should do all conversions between one unit with a prefix and another unit with a different prefix. Just convert the original unit to the base unit (like we converted kg to g ), and then convert from the base unit to the unit you need (like we converted from g to mg ).

## Comprehension Check

11. Convert 7.4 kg into cg .
12. Convert $11,230 \mathrm{~mL}$ into kL .

## Measuring Time

Sometimes, chemists need to know how long it takes for something to happen. That means they must measure the passage of time. Thankfully, the base metric unit for time is something you already know: seconds. Because it is a metric unit, of course, we can use prefixes to modify it, so when something happens incredibly quickly, very little time passes, so time might be measured in milliseconds. If something happens very slowly, lots of time passes, so time might be measured in kiloseconds.

What about minutes, hours, days, etc.? Those are valid units for time, but they are not metric. We can, of course, convert between those units and seconds. I will expect you to know that there are 60 seconds in a minute, 60 minutes in an hour, and 24 hours in a day. That way, no matter what time unit you are given, you can convert it into the metric unit of seconds.

## Density

Now that you know how to measure mass and volume, I want to cover one more thing before we finish this chapter: density.

Density - A measure of how tightly-packed the matter in a substance is
Consider, for example, the spinning tops pictured below. The one on the left is made of plastic, and the one on the right is made of lead. They are both similar in size and shape, but if you were to pick up both


The top on the left is made of plastic, and the one on the right is made of lead. Despite the fact that they are similar in size and shape, the top on the right has more mass, because lead is more dense than plastic. of them, which would feel heavier? Most people would know without touching them that the lead top would feel heavier. Why is that? From a scientific point of view, we would say it's because matter is more tightlypacked in lead than it is in plastic. As a result, an object made of lead has more mass (and therefore feels heavier) than an object of similar size and shape that is made of plastic.

This is what density is all about. It tells you how much mass is packed into a certain volume. To calculate the density of an object, then, all you have to do is measure its mass and volume. When you divide the latter into the former, you have the object's density:

$$
\begin{equation*}
\text { density }=\frac{\text { mass }}{\text { volume }} \tag{1.1}
\end{equation*}
$$

Now let's think about the units density will have. Equation (1.1) tells us that to get density, you divide mass by volume. That means the unit for density will be a mass unit divided by a volume unit. Thus, units like $\mathrm{g} / \mathrm{cm}^{3}, \mathrm{~g} / \mathrm{mL}, \mathrm{kg} / \mathrm{L}$, and slugs $/ \mathrm{gallon}$ are all acceptable density units. In case you forgot, slugs measure mass in English units (I just had to work my favorite mass unit into the discussion!). Chemists typically measure mass in grams and volume in $\mathrm{mL}\left(\mathrm{or} \mathrm{cm}^{3}\right)$, so the first two density units listed above are the most common ones you will see in chemistry.

## Example 1.10

The mass of a gold object is 38.6 kg , and its volume is 2.00 L . What is the density of the object in grams per mL ?

According to Equation (1.1), we just divide mass by volume to get density. However, the mean author of this book wants you to give the answer in grams per mL . Therefore, before we divide, we have to convert the mass and volume into the units we need.

$$
\begin{gathered}
\frac{38.6 \mathrm{~kg}}{1} \times \frac{1,000 \mathrm{~g}}{1 \mathrm{~kg}}=38,600 \mathrm{~g} \\
\frac{2.00 \pm}{1} \times \frac{1 \mathrm{~mL}}{0.001 \mathrm{I}}=2.00 \times 10^{3} \mathrm{~mL}
\end{gathered}
$$

Now we can use Equation (1.1):

$$
\begin{gathered}
\text { density }=\frac{\text { mass }}{\text { volume }} \\
\text { density }=\frac{38,600 \mathrm{~g}}{2.00 \times 10^{3} \mathrm{~mL}}=19.3 \frac{\mathrm{~g}}{\mathrm{~mL}}
\end{gathered}
$$

The density of the golden object, then, is $19.3 \mathrm{~g} / \mathrm{mL}$.

Density is a useful concept for many reasons, one of the most obvious being that it helps us identify substances. Generally speaking, each substance has its own density. So if something is really made of gold, it should have a density of $19.3 \mathrm{~g} / \mathrm{mL}$. Look at the picture on the right. If you found that in a stream, what would you think you found? Most people would think they'd found gold, because it looks a lot like gold. However, if you measured its mass and volume and then used Equation (1.1), you would find that its density is $4.9 \mathrm{~g} / \mathrm{mL}$. That's a lot less than the density of gold. What does that tell you? The substance in the picture is not gold. It is, in fact, a chemical called "iron pyrite," which is


This might look like gold, but its density is much too low to be gold. It is a chemical called "iron pyrite," or "fool's gold."

There are other uses for the concept of density. Consider, for example, an irregularly-shaped object, like the iron pyrite pictured on the previous page. It would be difficult to measure its volume. However, if you know it is iron pyrite, you know that its density is $4.9 \mathrm{~g} / \mathrm{mL}$. Given that fact, if you simply measured its mass (an easy thing to do), you could figure out its volume using Equation (1.1), as shown below.

## Example 1.11

The mass of a sample of iron pyrite is 1.32 kg . What is its volume? (The density of iron pyrite is $4.9 \mathrm{~g} / \mathrm{mL}$ ).

Notice that the problem gives you the mass in kg , but the density in $\mathrm{g} / \mathrm{mL}$. If we want to use these two pieces of information to solve the problem, we have to get them into consistent units. Thus, I have to either convert the mass to grams or the density into $\mathrm{kg} / \mathrm{mL}$. The first option seems a lot easier than the second, so let's do that:

$$
\frac{1.32 \mathrm{~kg}}{1} \times \frac{1,000 \mathrm{~g}}{1 \mathrm{~kg}}=1,320 \mathrm{~g}
$$

Now we can use Equation (1.1):

$$
\text { density }=\frac{\text { mass }}{\text { volume }}
$$

Of course, we need to do something to this equation, because we want to calculate the volume. So we need an equation that starts "volume $=$." How do we do that? We use algebra. If we multiply both sides of the equation by volume, it cancels on the left and appears on the right:

$$
\begin{gathered}
\text { volume } \cdot \text { density }=\frac{\text { mass }}{\text { volume } \cdot \text { density }=\text { mass }} \\
\text { verne }
\end{gathered}
$$

Now we can divide both sides by density, which is the same as multiplying by $1 /$ density:

$$
\begin{gathered}
\text { volume } \cdot \frac{1}{\text { density }}=\text { mass } \cdot \frac{1}{\text { volume }=\frac{\text { mass }}{\text { density }}}
\end{gathered}
$$

Now we can just plug in the numbers:

$$
\text { volume }=\frac{1,320 \mathrm{~g}}{4.9 \frac{\mathrm{~g}}{\mathrm{~mL}}}=270 \mathrm{~mL}
$$

Notice what happened to the units. Grams cancelled, because one was on top of the fraction, while the other was on the bottom. The $1 / \mathrm{mL}$ left on the bottom, then, turned into mL , because the inverse of $1 / \mathrm{mL}$ is mL . Of course, since volume is measured in mL , that makes sense. This is one way you can check whether or not you have solved a problem correctly. The units should work out to be the right unit for the measurement you are trying to figure out. Also, note that $4.9 \mathrm{~g} / \mathrm{mL}$ has only two significant figures, so the answer can have only two. The volume, then, is $\underline{270 \mathrm{~mL}}$.

## Comprehension Check

13. A chunk of copper has a volume of 20.0 mL . What is its mass? (The density of copper is $8.96 \mathrm{~g} / \mathrm{mL}$ )
14. The density of silver is $10.49 \mathrm{~g} / \mathrm{mL}$. What is the volume of a $9.43-\mathrm{kg}$ sample?

## More on Density

Density is so useful, I want you to have a bit more experience with it before you finish this chapter. Perform the following experiment.

## Experiment 1.3: Density

Materials

- A $50-\mathrm{mL}$ graduated cylinder
- A mass scale
- A medicine dropper
- A $250-\mathrm{mL}$ beaker
- Water
- Cooking oil
- Table salt in a salt shaker
- Safety goggles (Wear them throughout the experiment.)


## Instructions

1. Your mass scale runs on batteries, but they are probably covered with plastic. Pull them out, get rid of the plastic, and put them back in so the scale is ready to go.
2. Push the power button to turn on the scale. This button is also the tare (tair), which allows you to set the mass to zero. That way, when you put something on the scale, it reads only the mass of what you put there. There is also a button that says "unit," "mode," or "M." Press that button until the readout has a " $g$ " at the end, to indicate you are measuring grams.
3. If the readout on the scale doesn't say " 0.0 g ," press the tare again to make it read " 0.0 g ."
4. Put your graduated cylinder on the scale and read its mass. Record the measurement.
5. Remove the graduated cylinder from the scale and add some water to the graduated cylinder so it is near the $50-\mathrm{mL}$ mark. Then use the medicine dropper to add water until the bottom of the meniscus is right at the $50-\mathrm{mL}$ mark, which means the volume is 50.0 mL .
6. If the readout on the scale doesn't say " 0.0 g ," press the tare again to make it read " 0.0 g ."
7. Put the graduated cylinder back on the mass scale so it stands upright and doesn't spill any water.
8. Read the mass of the graduated cylinder and 50.0 mL of water. Record the measurement.
9. Subtract the mass of graduated cylinder (what you measured in step 4) from the mass of the graduated cylinder and 50.0 mL of water (what you measured in the previous step). This gives you the mass of 50.0 mL of water. Write that down. What you just did is called weighing by difference. You measured the mass of the empty cylinder and then the mass of the cylinder plus 50.0 mL of water. The difference between those two numbers was the mass of just the 50.0 mL of water.
10. Divide the mass of the water by the volume $(50.0 \mathrm{~mL})$ to get the density of the water.
11. Empty the graduated cylinder and get rid of as much water as you can.
12. Repeat steps 3-10, using oil instead of water.
13. Look at the two densities. In which liquid is matter packed more tightly?
14. Fill the $250-\mathrm{mL}$ beaker halfway with water.
15. Pour all the oil that is in the graduated cylinder into the $250-\mathrm{mL}$ beaker as well.
16. Notice that the oil sits on top of the water.
17. Shake lots of salt onto the oil that is floating on top of the water.
18. Watch through the side of the beaker and notice what happens after a while. In your notebook, describe or draw what you see. Feel free to shake more salt onto the oil to see more of the effect.
19. Clean up your mess, turn off the scale, and put everything away. Use lots of dish soap and warm water to clean out the graduated cylinder, beaker, and medicine dropper.

In the experiment, you found that water had a higher density than oil. This, of course, explains why oil floats on water. Remember, density tells you how tightly-packed the matter in a substance is. The matter in oil, then, is not as tightly-packed as the matter in water. As a result, the oil cannot push its way through the water, so it stays on top of the water.

But what happened when you added salt to the oil layer? You should have seen drops of oil sink down into the water. When those drops of oil hit the bottom of the glass, however, they should have risen back up to the top of the water layer. Why? Well, salt is more dense than both water and oil. As a result, it was able to shove its way down through the oil layer. When it reached the boundary between the oil and water layer, it continued to shove its way through, but some oil clung to the salt. The density of the salt plus the drop of oil was still greater than water, so the oil that clung to the salt fell through the water layer along with the salt.

When the oil and salt hit the bottom of the beaker, the salt then sank to the very bottom of the oil. At that point, it mixed with water and dissolved into the water. In other words, it left the oil drop. Once the salt left the oil drop, the oil drop had to float back up to the surface of the water, because it had no salt to increase its density.


This ship floats because even though it is made of steel and carrying a lot of cargo, its density is lower than the density of water.

This is another important thing you can learn from density - less dense substances tend to float on more dense substances. A huge ship, like the one pictured on the left for example, is made of steel. A block of steel sinks, because the density of steel is much greater than the density of water. However, a ship is made with an outer layer of steel and a lot of empty space inside. If you measure the total mass of the ship and divide by the total volume the ship occupies, you will find that the density is significantly less than the density of water, so the ship floats.

## Comprehension Check

15. The mass of a full can of Coke is 394 g . Because it uses an artificial sweetener, the mass of a full can of Diet Coke is lower: only 355 g . The volume of both cans (including the outside of the can) is 359 mL . Will either can float in water when it is full? (The density of water is $1.0 \mathrm{~g} / \mathrm{mL}$.)

You have completed the first chapter of your chemistry course! Now it's time to make sure you understand all that you learned. Answer all of the questions in the review you will find on the last two pages of this chapter. This should get you ready for the test. If you need more questions to prepare you for the test, you will find more on the course website.

## Solutions to the "Comprehension Check" Questions

1. The pencil is 7.41 inches long and 18.82 centimeters long. That means it takes more centimeters to make up the length of the pencil than it does inches. Thus, a centimeter must be smaller than an inch. This means a 1-inch-long object is longer than a 1-centimeter-long object.
2. (a) The first two zeroes are not significant, because they are not between significant figures or at the end of the number. The " 1 " and " 2 " are significant by Rule \#1, which makes the zero in between them significant by Rule \#2. Rule \#3 says the last zero is significant, so there are four significant figures.
(b) Rule \#1 tells us that the " 1, ," "2," and " 7 " are all significant. Since the two zeroes are in between the " 2 " and " 7 ," Rule \#2 says they are significant. That means five significant figures.
(c) Rule \#1 tells us the " 6 " and " 9 " are significant, which makes the zero in between them significant by Rule \#2. The last three zeroes are at the end of the number, but they are not to the right of the decimal. Thus, they are not significant. That means there are three significant figures.
(d) Rule \#1 tells us that the " 8 ," " 9 ," " 1, ," and " 4 " are significant. Rule $\# 2$ tells us the zero in between the " 1 " and " 4 " is significant. Rule \#3 tells us that the last two zeroes are significant. That means there are seven significant figures.
3. First, we just add the numbers:

$$
3.1 \text { in }+8.991 \text { in }=12.091 \text { in }
$$

Since 3.1 has its last significant figure in the tenths place, it is the least precise. Thus, the answer must be reported to the tenths place: 12.1 in.
4. First, we multiply the numbers:

$$
45.5 \mathrm{~cm} \cdot 9 \mathrm{~cm}=409.5 \mathrm{~cm}^{2}
$$

Once again, just as $x \cdot x=x^{2}, \mathrm{~cm} \cdot \mathrm{~cm}=\mathrm{cm}^{2}$. The first number has three significant figures, but the second has only one. That means the answer can have only one significant figure, so it is $400 \mathrm{~cm}^{2}$.
5. We start by taking the measurement and putting it over 1 :

$$
\frac{33.56 \mathrm{~cm}}{1}
$$

Now we multiply by a fraction made from the conversion relationship ( $2.54 \mathrm{~cm}=1 \mathrm{in}$ ), but the fraction must have centimeters on the bottom so it cancels:

$$
\frac{33.56}{1} \times \frac{1 \mathrm{in}}{2.54 \mathrm{~m}}=13.21 \mathrm{in}
$$

Since the original measurement has four significant figures, the answer must have four. That's why the answer is 13.21 in .
6. First, we get the conversion relationship by replacing the prefix (milli) with its meaning (0.001):

$$
1 \mathrm{~mm}=0.001 \mathrm{~m}
$$

Notice how the left side of the conversion relationship contains the unit with the prefix ( mm ), while the right side has the prefix's definition ( 0.001 ) followed by the base unit $(\mathrm{m})$. Now we put the original measurement over 1 to make it a fraction and multiply by the conversion relationship so that mm cancels:

$$
\frac{17.9 \mathrm{~m}}{1} \times \frac{0.001 \mathrm{~m}}{1 \mathrm{~mm}}=0.0179 \mathrm{~m}
$$

Since the original measurement has three significant figures, the answer must have three. That's why the answer is 0.0179 m .
7. a. To get the decimal to the right of the " 5 ," we must move it 10 places to the left. The number is large, so the exponent is positive. That gives us $5.3 \times 10^{10}$. However, the problem said there needs to be five significant figures, so the answer is $5.3000 \times 10^{10}$.
b. To get the decimal to the right of the " 9 ," we need to move it three places to the right. The number is small, so the exponent has to be negative. That gives us $\underline{9.4230 \times 10^{-3}}$. Note the last zero needs to be there because the problem asked for five significant figures.
8. a. The exponent is negative, so this is a small number. That means we move the decimal to the left. The exponent is -8 , so we move it 8 places to the left. That gives us $\underline{0.00000008612}$.
b. The exponent is positive, so this is a big number. That means we move the decimal to the right. Since the exponent is 4 , we move it four places to the right, giving us $\underline{69,650}$.
9. This one isn't bad. We know that $\mathrm{cm}^{3}$ and mL are the same, so we just have to convert 7.4 liters into mL .

$$
\frac{7.4 \pm}{1} \times \frac{1 \mathrm{~mL}}{0.001 \pm}=7,400 \mathrm{~mL}
$$

That means the volume is also $7,400 \mathrm{~cm}^{3}$.
10. We don't know a relationship between liters and $\mathrm{m}^{3}$, but we do know that a mL is the same as a $\mathrm{cm}^{3}$. That means we can convert liters to mL , which is also $\mathrm{cm}^{3}$. Then, we can convert from $\mathrm{cm}^{3}$ to $\mathrm{m}^{3}$.

$$
\frac{11.7 \pm}{1} \times \frac{1 \mathrm{~mL}}{0.001 \mathrm{I}}=11,700 \mathrm{~mL}
$$

That means the volume is also $11,700 \mathrm{~cm}^{3}$. Now we use the cm to m conversion relationship ( $1 \mathrm{~cm}=0.01$ m ) in a fraction. However, remember that this is $\mathrm{cm}^{3}$. Once we make the fraction, we have to cube it, so that $\mathrm{cm}^{3}$ cancels:

$$
\frac{11,700 \mathrm{~cm}^{3}}{1} \times\left(\frac{0.01 \mathrm{~m}}{1 \mathrm{~cm}}\right)^{3}
$$

When you cube a fraction, you cube everything inside. That means you cube 0.01 to make it 0.000001 , you cube m to make it $\mathrm{m}^{3}$, you cube 1 to make it 1 , and you cube cm to make it $\mathrm{cm}^{3}$. That turns the equation into:

$$
\frac{11,700 \mathrm{~m}^{3}}{1} \times \frac{0.000001 \mathrm{~m}^{3}}{1 \mathrm{~m}^{3}}=0.0117 \mathrm{~m}^{3}
$$

This tells us that 11.7 liters is the same as $0.0117 \mathrm{~m}^{3}$.
11. We have to convert to the base unit first, which means going from kg to g :

$$
\frac{7.4 \mathrm{~g}}{1} \times \frac{1,000 \mathrm{~g}}{1 \mathrm{~g}}=7,400 \mathrm{~g}
$$

Now we can convert to cg, remembering that " $c$ " stands for "centi," which means "0.01."

$$
\frac{7,400 \mathrm{~g}}{1} \times \frac{1 \mathrm{cg}}{0.01 \mathrm{~g}}=740,000 \mathrm{~g}
$$

That means 7.4 kg is the same as $740,000 \mathrm{cg}$.
12. These measurements are for volume, not mass, but the method is the same. We convert from the prefix unit to the base unit $(\mathrm{mL}$ to L$)$ and then from the base unit to the other prefix unit $(\mathrm{L}$ to kL$)$ :

$$
\begin{aligned}
& \frac{11,230 \mathrm{~mL}}{1} \times \frac{0.001 \mathrm{~L}}{1 \mathrm{~mL}}=11.23 \mathrm{~L} \\
& \frac{11.23 \pm}{1} \times \frac{1 \mathrm{~kL}}{1,000 \pm}=0.01123 \mathrm{~kL}
\end{aligned}
$$

The answer, then, is $\underline{0.01123 \mathrm{~kL}}$.
13. We have volume and density and want to get the mass. This is easy, since Equation (1.1) relates all three of them. First, however, we must check the units. The volume is in mL , and the density is in mL , so that works, because the common unit is mL in both cases.

$$
\text { density }=\frac{\text { mass }}{\text { volume }}
$$

To get mass, we just multiply both sides by volume:

$$
\begin{gathered}
\text { volume } \cdot \text { density }=\frac{\text { mass }}{} \cdot \\
\text { volume } \cdot \text { density }=\text { mass }
\end{gathered}
$$

In algebra, we can always turn an equation around, so:

$$
\text { mass }=\text { volume } \cdot \text { density }=20.0 \quad 8.96 \frac{\mathrm{~g}}{\mathrm{~mL}}=179 \mathrm{~g}
$$

Notice that mL cancel, leaving only grams, which is good since we were calculating mass. Also, 20.0 has three significant figures, as does 8.96 , so the answer must also have three, which is why it is 179 g .
14. We have density and mass (the unit kg measures mass), and we want to determine volume. That means we can use Equation (1.1). However, we have to check units. The density is in $\mathrm{g} / \mathrm{mL}$, but the mass is in kg , so we need to convert. Let's convert kg to g .

$$
\frac{9.43 \mathrm{~kg}}{1} \times \frac{1,000 \mathrm{~g}}{1 \mathrm{~kg}}=9,430 \mathrm{~g}
$$

Now we can use Equation (1.1):

$$
\text { density }=\frac{\text { mass }}{\text { volume }}
$$

Multiplying both sides by volume gives us:

$$
\begin{gathered}
\text { volume } \cdot \text { density }=\frac{\text { mass }}{} \cdot \\
\text { volume } \cdot \text { density }=\text { mass }
\end{gathered}
$$

Now we can divide both sides by density, which is the same as multiplying by 1 /density:

$$
\begin{gathered}
\text { volume } \cdot \frac{1}{\text { density }}=\text { mass } \cdot \frac{1}{\text { volume }=\frac{\text { mass }}{\text { density }}}
\end{gathered}
$$

Now we can just plug in the numbers:

$$
\text { volume }=\frac{9,430 \frac{\mathrm{~g}}{\mathrm{o}}}{10.49 \frac{\frac{\mathrm{~g}}{\mathrm{~g}}}{\mathrm{~mL}}}=899 \mathrm{~mL}
$$

The units work out properly, and since 9,430 has three significant figures and 10.49 has four, the answer must have three significant figures. Thus, the volume is 899 mL .
15. To determine whether or not something floats, we have to determine its density. We are given mass and volume in both cases, so it is pretty easy:

$$
\text { density of the full Coke can }=\frac{\text { mass }}{\text { volume }}=\frac{394 \mathrm{~g}}{359 \mathrm{~mL}}=1.10 \frac{\mathrm{~g}}{\mathrm{~mL}}
$$

The answer was actually $1.09749 \ldots$, but the measurements have three significant figures, so the answer needs three. That means I have to drop the " 7 ," which means rounding the " 9 " to a " 10 ." I need to keep
the zero, though, because I need three significant figures. Since the density of a full Coke can is greater than that of water, the full Coke can will not float in water. For the Diet Coke can:

$$
\text { density of the full Diet Coke can }=\frac{\text { mass }}{\text { volume }}=\frac{355 \mathrm{~g}}{359 \mathrm{~mL}}=0.989 \frac{\mathrm{~g}}{\mathrm{~mL}}
$$

This is less than the density of water, so the Diet Coke can will float. You can actually do this in a sink full of water. Make sure to use 12 -ounce cans, and make sure no air bubbles get trapped under the cans when you put them in the water. The water should be on the cold side, since the density of a liquid depends on its temperature. If you do this in a sink that is deeper than the cans are tall, you will see the Coke can sink to the bottom. The Diet Coke will become almost fully submerged, but it will not completely sink.

## Sample Calculations for Experiment 1.1

Width of the book: $14.00 \mathrm{~cm}, 5.51$ in
Centimeters divided by inches: $14.00 \mathrm{~cm} \div 5.51$ in $=2.54 \frac{\mathrm{~cm}}{\text { in }}$
(14.00 has four significant figures, while 5.51 has three. The answer, therefore, can have only three. Also, when you divide centimeters by inches, the result is a fraction with centimeters on the top and inches on the bottom. This is generally read as "centimeters per inch.")

Length of the book: $21.55 \mathrm{~cm}, 8.50$ inches
Centimeters divided by inches: $21.55 \mathrm{~cm} \div 8.50$ inches $=2.54 \frac{\mathrm{~cm}}{\mathrm{in}}$
Width of the spine: $1.25 \mathrm{~cm}, 0.50$ inches
Centimeters divided by inches: $1.25 \mathrm{~cm} \div 0.50$ inches $=2.5 \frac{\mathrm{~cm}}{\text { in }}$
( 1.25 has three significant figures, while 0.50 has only two. The answer, therefore, can have only two.)

## Sample Calculations for Experiment 1.2

Height of the pill bottle: 10.25 cm
Diameter of the pill bottle: 2.63 cm
Radius of the pill bottle: $2.63 \mathrm{~cm} \div 2=1.32 \mathrm{~cm}$ (The " 2 " is exact, because the radius is exactly half of the diameter. Thus, it has infinite significant figures. The answer, then, needs to have the same number of significant figures as the original measurement, which is three.)

Volume of the pill bottle: $\mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}=(3.14) \cdot(1.32 \mathrm{~cm})^{2} \cdot(10.25 \mathrm{~cm})=56.1 \mathrm{~cm}^{3}$ (Remember that $\pi$ is an irrational number which tells you the ratio of a circle's diameter to its radius. It is equal to $3.14159265 \ldots$. You could have used more significant figures for $\pi$, but since the measurement of 1.32 cm limits the answer to three significant figures, I just used three significant figures in the value for $\pi$.)

Number of complete graduated cylinders dumped into the sink: 1
Volume of the water in the final graduated cylinder: 6.0 mL
Volume of water in the pill bottle: $6.0 \mathrm{~mL}+50.0 \mathrm{~mL}=56.0 \mathrm{~mL}$. (Both measurements have their last significant figure in the tenths place, so the answer must be reported to the tenths place.)

Although the volumes aren't exactly equal, they are very similar ( $56.1 \mathrm{~cm}^{3}$ as compared to 56.0 mL ). Remember, there is always some error in the last significant figure, so small differences in the last significant figure can be expected, even when two measurements should be the same. From a scientific point of view, then, these measurements are really equal. That's because $\mathrm{cm}^{3}$ and mL are equivalent.

## Sample Calculations for Experiment 1.3

Mass of the graduated cylinder: 29.5 g
Mass of the graduated cylinder plus water: 79.4 g
Mass of the water: $79.4 \mathrm{~g}-29.5 \mathrm{~g}=49.9 \mathrm{~g}$
(Each mass has its last significant figure in the tenths place, so the answer must as well.)
Volume of the water: 50.0 mL
Density of the water: $49.9 \mathrm{~g} \div 50.0 \mathrm{~mL}=0.998 \mathrm{~g} / \mathrm{mL}$
( 50.0 mL has three significant figures, as does 49.9 g , so the answer must also have three.)
Mass of the graduated cylinder: 29.5 g
Mass of the graduated cylinder plus oil: 75.6 g
Mass of the oil: $75.6 \mathrm{~g}-29.5 \mathrm{~g}=46.1 \mathrm{~g}$
(Each mass has its last significant figure in the tenths place, so the answer must as well.)
Volume of the oil: 50.0 mL
Density of the oil: $0.922 \mathrm{~g} / \mathrm{mL}$
( 50.0 mL has three significant figures, as does 46.1 g , so the answer must also have three.)

## Review

1. Define the following terms:
a. Matter
b. Unit
c. Significant figure
d. Weight
e. Mass
f. Density
2. What are the base metric units used to measure length, mass, and time?
3. You are reading a scientist's lab notebook and see a measurement of 14.5 mL . What was the scientist measuring: length, mass, volume, or time?
4. You are measuring the volume of an object using a scale that is marked off with lines that represent 10 mL each. To what level of precision (one mL , tenths of an mL , hundredths of an mL , etc.) should you report your measurement?
5. An object has a mass of 123.4 kg , which is the same as 8.456 slugs. Which measures more mass: 1 slug or 1 kg ?
6. A box has a volume of $1.01 \mathrm{~m}^{3}$. Two students measure the box. The first says the volume is $1 \mathrm{~m}^{3}$, while the second says the volume is $1.23 \mathrm{~m}^{3}$. Assuming they are reporting the correct number of significant figures for the measuring devices used, which student used the more precise device? Which student provided the more accurate answer?
7. How many significant figures are in the following measurements?
a. $1.06 \times 10^{4} \mathrm{~mL}$
b. $12,000 \mathrm{~cm}$
c. 0.0340 kg
d. $1.0 \times 10^{1}$ in
8. When you put ice cubes in a glass of water, the ice cubes float. What does that tell you about the density of ice compared to the density of water?
9. When a figure is significant, does that mean it is mathematically important?
10. Two students are measuring the mass of an object. One reports his answer as 4.56 g , while the other reports her answer as 4.58 g . The teacher gives each student $100 \%$ credit. How can they both be right?
11. A carpet-layer measures a room to be 4.6 m long and 3.2 m wide. He multiplies the two measurements and reports the area of the room to be 14.72 . What two things are wrong with his answer?
12. What is the correct answer to the following equation?

$$
21.0234 \mathrm{~g}-12 \mathrm{~g}
$$

13. A U.S. dime has a diameter of 17.9 mm . What is the diameter in m ?
14. A soup can has a mass of 490 grams. What is its mass in kg?
15. The volume of a soccer ball is $6,080 \mathrm{~mL}$. How many kL is that?
16. An object has a length of 15.0 inches. What is its length in meters? ( $2.54 \mathrm{~cm}=1$ inch)
17. Convert the following measurements into scientific notation with 4 significant figures each.
a. 0.0001214 kg
b. $34,500 \mathrm{~m}$
c. $123,500,000 \mathrm{mg}$
d. 0.01010 km
18. Convert the following measurements into decimal notation.
a. $1.2 \times 10^{-2} \mathrm{~km}$
b. $7.82 \times 10^{7} \mathrm{~mL}$
c. $9.1 \times 10^{-8} \mathrm{~kg}$
d. $8.912 \times 10^{6} \mathrm{~m}$
19. A bowl has a volume of 1.1 L . What is its volume in cubic centimeters?
20. A sample of liquid has a volume of 143.6 L . What is the volume in $\mathrm{m}^{3}$ ?
21. Aluminum has a density of $2.70 \mathrm{~g} / \mathrm{mL}$. What is the volume of an aluminum block that has a mass of 55.67 kg ?
22. A sample of what looks like silver has a mass of 1.7 kg and a volume of 0.164 liters. Is it really silver? (The density of silver is $10.49 \mathrm{~g} / \mathrm{mL}$.)


This Erlenmeyer flask contains a heterogeneous mixture, which you will learn about in Chapter 2.

